

## Railway Ballast Settlement: A New Predictive Model

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### Abstract

By means of a detailed parametric study of the settlement of ballast material performed on a full-scale track model, a predicting model for ballast settlement under cyclic loading has been developed. The model is based on track parameters: axle load, train speed and the initial mechanical state of ballast. A light dynamic penetrometer allows the characterization of the latter. Several loading tests, were performed, together with a characterization of the initial state of ballast close to the sleepers. Three hundred and sixty curves of settlement were obtained. The results show a settlement evolution in three stages (short, medium and long term settlement) depending on the initial conditions of the material and the intensity of the vibration. The settlement models reviewed in literature (Shenton, Hettler or Tom and Okley model) are not able to describe the entire evolution of the settlement curves. By developing a model based on the Chicago's density relaxation law, it is possible to estimate the evolution of settlement from the loading parameters and the initial mechanical state of the ballast material. From the database and studies of the Chicago experiments two governing parameters, can be identified, which represent the intensity of loading and cone resistance in coarse granular material. From these parameters it is possible to identify three functions which give the three parameters of the proposed model of the settlement. The proposed model describes the three stages in the settlement evolution. This model, si selected, for the results obtained in the global analysis of error by the MSE method. With this model, we obtain a prediction of settlement evolution with an error less than 10% for almost all experimental data. As a conclusion a method is proposed which has been tested on a track and which allows one to estimate the settlement of the sleepers by considering train traffic and cone penetration resistance in order to obtain an average settlement curve and a probability to reach a threshold value. This is a first step towards a general framework for the evaluation of the geometric potential degradation of railway tracks, which is a major issue for the cost reduction of maintenance operations on railway tracks.

**Keywords:** ballast settlement, full-scale track model, settlement model, light penetrometer test.

## 1 Introduction

Railway ballast is a granular material, generally comprises large, angular particles of typical size ranging between 25 and 50 mm. The main functions of railway ballast are to reduce pressures from the sleeper bearing area to acceptable levels at the surface of the subgrade soil, to facilitate maintenance operations for the re-establishment of track riding quality, and to provide rapid drainage [1].

In this material, plastic deformations induced by traffic loading, lead to the settlement and deterioration of track geometry [2, 3, 4, 5]. This settlement is the result of the granular compaction under cyclic loading. This track deterioration reduces the safety and comfort levels of traffic. In the other hand, the profile correction demands the operation of expensive equipment such as tamping process.

The challenge to the railway industry is to improve efficiency and stability of track while reducing the track maintenance cost. It is therefore crucial to understand the mechanical behaviour of ballast layer for predicting the track deterioration. Over the years, several but inconclusive studies are developed to understand the physical mechanisms behind the irreversible deformation in coarse granular materials [6]; although explaining these phenomena have an important industrial interest.

In this paper we report on a detailed parametric study of the settlement for ballast material with the aim of developing a predicting model of settlement for this kind of material under cyclic loading. This work was meant to provide microstructural data allowing for a better understanding of our experimental investigation on a full-scale track model composed of two monobloc sleepers submitted to different loading in terms of frequency and loading. A major difficulty in granular materials is to be able to characterize its initial state. There is presently no established method for the characterization of the mechanical state for coarse granular materials. In this respect, we used a light cone penetrometer test with a special device which is classically used on railway French network for preparing track renewal. We carried out several test with the characterization of the initial state of ballast. Our results show that the evolution of ballast settlement can be fitted by a phenomenological model based on the density relaxation law for the granular materials. Moreover, the parameters from this model can be obtained by the loading parameters and the initial characterization of the ballast state.

In the following, we describe the full-scale model used in our experiences as well as the protocol establish to carry out the loading tests. In section 3, we characterize the evolution of settlement from the experimental data and we identify this evolution, we compare the empirical models with the models based on density relaxation. Finally, in section 4 we show the precision of the chosen predicting model.

## 2 Experimental set-up

On a full-scale model track we carried out several experimental tests combinations, for different levels of loading frequency, which represents different values of train speed and axle load over a rigid subgrade. We carried out these experiences between June 2010 and March 2011, developing 15 loading tests combinations with 6 repetitions for each one. Finally we have obtained 360 settlement curves.

The track model is composed by two monoblock sleepers lying on a ballast layer of thick 0.35 m (Fig. 1). The ballast layer is confined into a wood box with dimensions 4.11 m  $\times$  1.5 m. In this track model, the subgrade is represented by a wooden plate. The sleepers are fixed each other by a rail section. A hydraulic cylinder applies the loading on the track by a sinus wave. With the aim to provides an uniform load distribution, a beam is fixed to the hydraulic cylinder applying the loading on the rails. Different load levels are simulated by varying the load peak and frequency, where the load peak simulate the axle load values and the values of frequency the train speed. The load peak takes the values: 194, 239 and 272 kN, while the values of frequency were comprises between 3.3 and 6 Hz. In this way, we can form 15 different configurations for load and frequency values, in order to carry out a parametric study.



Figure 1: The full-scale track model used in this study

For developing the loading test, first we evaluate the initial track conditions with the light dynamic penetrometer Panda [7]. For railway tracks, penetration tests are used to determinate parameters for predicting subgrade performance [1, 8]. The light penetrometer can be also used for the characterization of ballast layer in order to evaluate some indicator about ballast compaction [9, 10]. Indeed, initial density plays a major role in the evolution of compaction of granular materials subjected to cyclic loading [11, 12, 13]. We characterize with several Panda tests the initial mechanical state of ballast layer into our full-scale track model. The principle of light dynamic penetrometer Panda is to push into the material, by manual beating with a hammer, a cone tip of 2 cm<sup>2</sup> cross-section. For each blow, the depth of penetration and the driv-

ing variable energy are measured to calculate the dynamic cone resistance ( $q_d$ ) with the corresponding depth using the Dutch Formula (Eq. 1):

$$q_d = \frac{E}{A_{tip} \times e} \frac{M}{M + P} \quad (1)$$

where  $M$  is the mass of the hammer,  $P$  is the weight of the driven parts during impact (impact head, road and tip),  $E$  is the kinetic energy fed into the system during the impact,  $e$  is the penetration per blow and  $A_{tip}$  is the cross-section of the tip [14, 15].

After characterizing the initial state of the ballast layer, we start the cyclic loading on the track model, for different configurations of load and frequency. The loading is applied during  $10^4$  load cycles to study the variability of settlement until the steady state is reached.

During the loading process, we measure with 4 non-contact optical sensors the evolution of settlement as a function of the number of cyclic loads. Hence, we obtain 24 curves for each load combination (load peak and frequency). Finally we compare the settlement data with the loading factors and initial conditions for developing a settlement model.

### 3 Experimental results

During the analyse of the experimental results, we can identify an important variability of the response obtained for all curves in the same test configuration (same loading parameters). Thus, it is possible to obtain a mean curve in order to characterize the average evolution of settlement for each configuration (Fig. 2). To establish the upper and lower limits of settlement evolution for the various tests, we calculated the coefficient of variation ( $C_v$ ), i.e. the ratio between standard deviation and mean values, across the groups of curves. Fig. 2 shows an example where we can see the upper and lower limits (for  $\pm 1$  standard deviation) for the same level of stress.

For each couple load - frequency, the  $C_v$  values do not seem to depend on the loading levels. It is remarkable that for all groups of curves (curves obtained for the same load), we find a similar value of  $C_v = 0.3$  (Fig. 3). The coefficient of variation shows a peak in the first 100 cycles and then stabilized beyond 500 cycles and take a value around  $C_v = 0.3$ . The peak reflects the arrangement of the grains at the surface in contact with the sleeper, where the average value of the settlement is very small compared to its fluctuations. After this phase, the variability of settlement fall-off to reach a constant level throughout the cyclic loading. The constant value of the coefficient of variation means that the standard deviation increases proportionally to the value of the settlement. The variability in the response of ballast settlement highlights the stochastic nature for this phenomenon in coarse granular materials [16].

Another important point is the shape of the settlement evolution curve. We observe that the settlement curves have a particular shape composed by 3 stages (Fig. 4). The first stage is the initial compaction of ballast. This initial compaction is very quick,

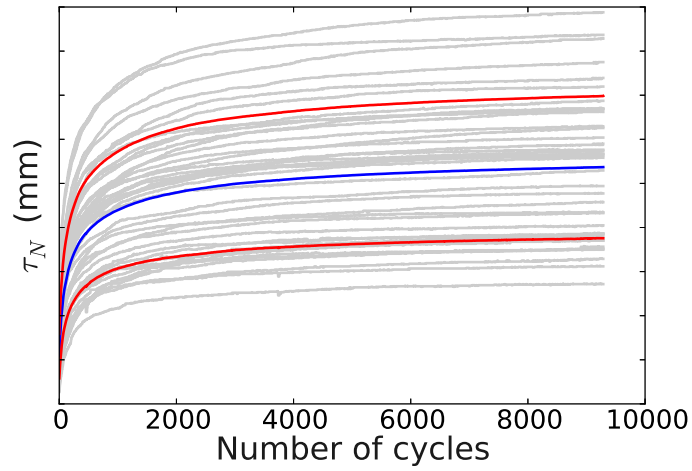


Figure 2: Settlement evolution as a function of the number of applied load for a curve group with a frequency of 3.3 Hz and a applied load of 194 kN. Blue curve shows the mean settlement and red curves show the upper and lower limits of settlement.

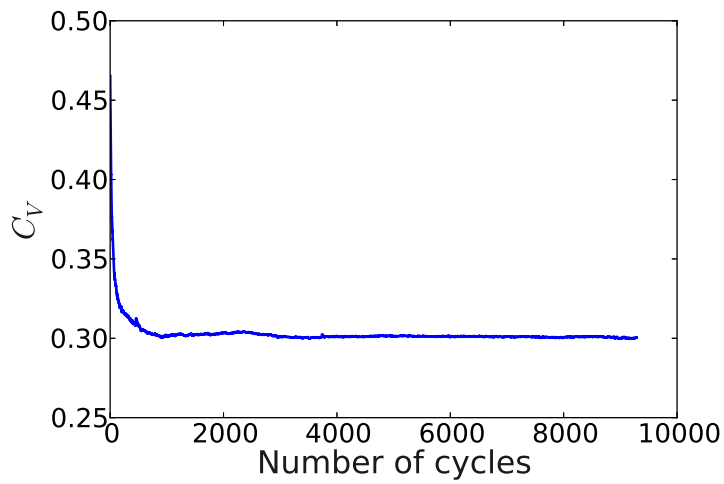


Figure 3: Coefficient of variation as a function of the number of applied load for a curve group with a frequency of 3.3 Hz and a applied load of 194 kN.

because it takes place on the first 200 cycles. This linear evolution is linked to the initial mechanical state of ballast. The second stage appears between 200 and 6000 cycles, we can call it "medium term behaviour". In this stage occur several rearrange-

ments in grains, under the effect of the cyclic loading and vibrations that produce sliding between the grains, with a gradual increase in the overall compactness of the system and a non-linear evolution of settlement. Finally, the last stage is a long-term behaviour observed beyond 6000 cycles. In this part of the curve, the compaction followed a very slow and almost linearly evolution with the number of cycles. This slow settlement can be attributed to rare events such as sudden rearrangements induced by erosion and rupture of grains.

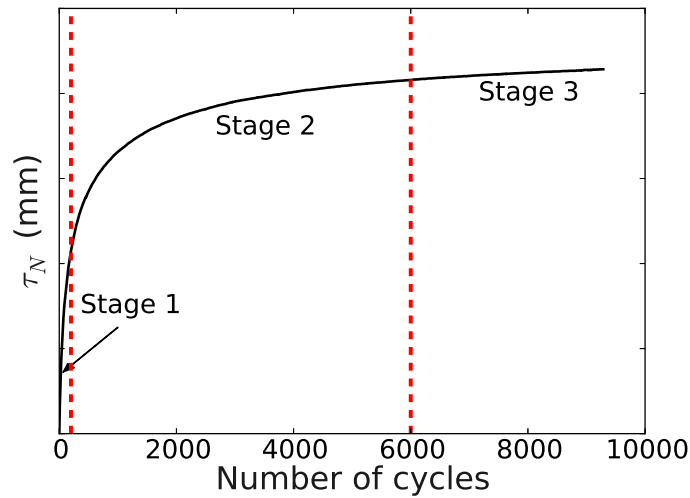


Figure 4: The three stages during the ballast settlement process.

## 4 Modelling of ballast settlement

With the aim to predict the evolution of ballast settlement, according to the evolution in three states, we have studied different proposed models of settlement, based on the axle load, vertical stress, subgrade stiffness and calibration constants.

### 4.1 Review of ballast settlement models

In this section, we try to identify an evolution model for the average ballast settlement. The main empirical models of settlement proposed in literature are summarized in Table 1.

This models takes into count the axle load  $Q$ , the stress under the sleeper  $p$ , the subgrade stiffness  $k$  and several calibration constants ( $\alpha$ ,  $\beta$  and  $\gamma$ ). These expressions do not describe quite well the entire evolution for our settlement curves as can be seen in Fig. 5. Shenton's expression fits well the beginning of the curve, but then there is an overestimation of settlement for a larger number of cycles. The expressions of

author	model
Sato	$\tau_N = \gamma[1 - e^{-\alpha N}] + \beta N$
Hecke	$\tau_N = \tau_0 + \alpha Q^3 N$
Hettler	$\tau_N = \alpha Q^{1.6}[1 + \beta \ln(N)]$
Shenton	$\tau_N = Q^5[\alpha N^{0.2} + \beta N]$
Thom & Oakley	$\tau_N = [\log N - 2.4]^2 \times [p/160] \times [47/k]$

Table 1: Ballast settlement models.

Sato and Hettler not show a better deal until 5000 cycles. Moreover, we have a lack of information about the physical meaning of the employed calibration constants. For this reason, in the next section we proposed a new model for ballast settlement.

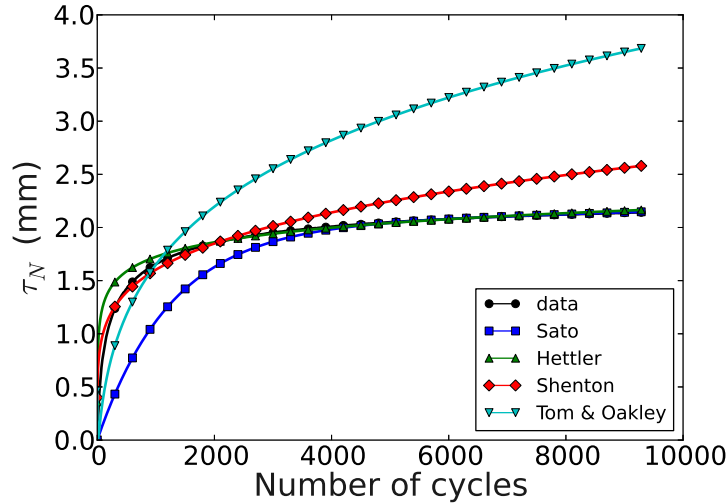


Figure 5: Comparison between the experimental data obtained for a test with a frequency of 3.3 Hz and applied load 194 kN, and the settlement models.

## 4.2 Formulation of a new settlement model

For establishing a ballast settlement model taking into account the granular nature of ballast, we adapted the density relaxation laws to study the settlement problem. These models are based in the fact that during the progressive compaction, when a void with the size of a particle is created, it is quickly filled by a new particle; voids with a particle size becomes more and more rare, and a large number of particles must be arranged to accommodate an additional particle in the system [17, 12]. Hence, the increment in the local density becomes slower as a function of time. We focus

in the main density relaxation laws: the Chicago's fit [11, 13, 12] and the Kohlraush-Williams-Watts law (KWW law) used by the Rennes group's work [18, 19, 20, 21, 22]. These laws were obtained by experimental analysis of the evolution of density for monodisperse spherical particles (glass beads) in a cylindrical tube under a series of external excitations, consisting of vertical shakes or "taps" applied to the container. The first relaxation law is an inverse-logarithmic law, described by (Eq. 2):

$$\rho(t) = \rho_f - \frac{\rho_f - \rho_0}{1 + B \ln(1 + t/t_0)} \quad (2)$$

where  $\rho_f$  is the value in the steady-state of the packing-fraction,  $\rho_0$  is the initial packing-fraction value,  $B$  is a fitting parameter and  $t_0$  is the characteristic time. In this relation, the only control parameter is the dimensionless acceleration  $\Gamma = a/g$  (the ratio between the tap peak acceleration and the gravity acceleration). The Rennes group's work found that the relaxation in compaction is better fitted by the KWW law, a stretched exponential (Eq. 3):

$$\rho(t) = \rho_f - (\rho_f - \rho_0) \exp[-(t/t_0)^\beta] \quad (3)$$

where  $t_0$  is the characteristic relaxation time and  $\beta$  is the stretching of the exponential. We adapt these equations 2 and 3 to our settlement problem. Thus we obtain:

$$\tau_N = \tau_\infty \left( 1 - \frac{1}{1 + B \ln(1 + \frac{N}{N_0})} \right) \quad (4)$$

$$\tau_N = \tau_\infty (1 - \exp[-(N/N_0)^\beta]) \quad (5)$$

According to the equations 4 and 5 the evolution of settlement  $\tau(N)$  as a function of the number of load cycles is described by the value of settlement in the steady-state  $\tau_\infty$  (asymptotic value), stretching parameters, and characteristic time in settlement, in our case, a number of cycles  $N_0$ .

For 360 curves of settlement, the parameters for these two models was identified, and then we try to predict the settlement evolution using the identified model parameters. The comparison between the obtained model parameters and loading parameters show that  $\tau_\infty$  and  $B$  depend on the normalized vibration intensity  $\Gamma = (A\omega^2)/(pd^2/m + g)$ , where  $A$  is the amplitude of the vibration,  $\omega$  is the angular frequency,  $p$  is the pressure under the sleeper,  $d$  is the average diameter of a particle,  $m$  is the average mass of a grain and  $g$  is the gravitational acceleration. For the calculation of  $\Gamma$  we added the acceleration of grains  $pd^2/m$ , in order to take into account the coarse granular nature of ballast during the vibration process. In the other hand, we find that the  $N_0$  parameter is linked to the initial density of the material. This parameter describes the initial slope of settlement, which is directly linked to the initial packing-fraction, qualified by the light penetrometer device.

Fig. 6 displays the comparison between the experimental data for a settlement test and the proposed models. Both models describe with a good agreement the settle-



ment evolution and the shape of the settlement curve; the three stages of settlement evolution from the three model parameters.

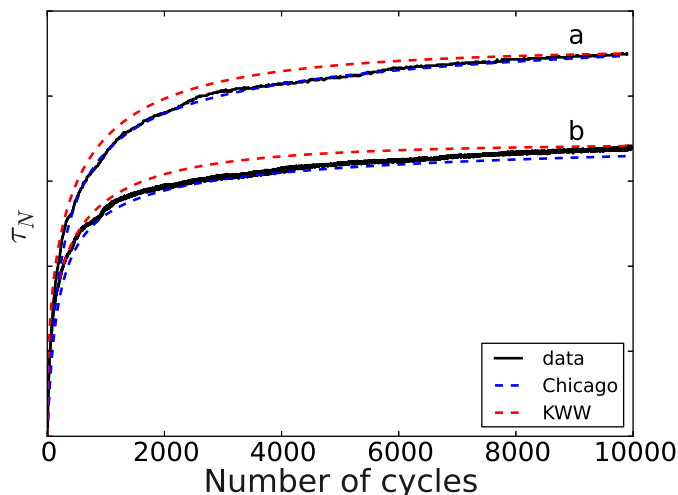


Figure 6: Comparison between the experimental data and the proposed models for two tests: (a) frequency of 4.5 Hz and applied load 239 kN; (b) frequency of 3.3 Hz and applied load 194 kN.

### 4.3 Model validation

In order to evaluate the quality of the prediction models, we calculate the mean square error (MSE) obtained between all the experimental and the predicted curves. The mean square error is defined by the Equation 6:

$$MSE(\hat{\theta}|\theta) = \langle (\hat{\theta} - \theta)^2 \rangle \quad (6)$$

where  $\hat{\theta}$  is the vector of the predicted values for each curve obtained by the proposed model and  $\theta$  is the vector of the measured values for each curve. In Fig. 7 we can see the cumulative distribution function for all the values obtained from the comparison between measured and predicted data (a total of 360 comparisons) for the two models. We note the model adapted from the Chicago's fit is more accurate for predicting settlement. In fact, we can see that 40% of the curves are predicted with an error less than 5% and we have 60% of cases with a prediction error less than 10%. This fact shows that the phenomenological model based from Chicago's fit is more adapted to predict the settlement evolution in coarse granular materials than KKW law.

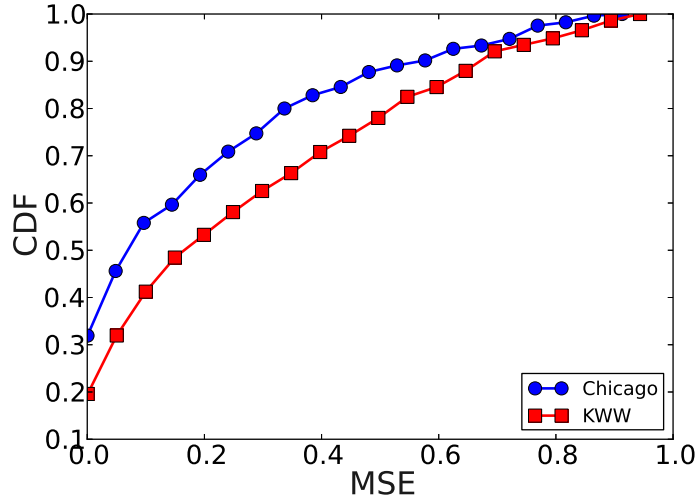


Figure 7: Cumulative distribution function of the mean square values for the two proposed models.

## 5 Concluding remarks

In this work, we had carried out a parametric study on a full-scale track model in order to develop a phenomenological model for predicting the settlement for ballast material under cyclic loading. The results show an important variability of the evolution of settlement curves, about 30%, for equal values of loading parameters (axle load, frequency). This fact shows the stochastic nature of ballast settlement. The results show as well a settlement evolution in three stages (short time, medium and long term settlement), depending on initial conditions of material and the intensity of vibration. By developing a model based on the Chicago's density relaxation law, we are able to estimate the evolution of settlement from loading parameters and the initial mechanical state of ballast. We chose this model for the results obtained in the global analysis of error by the MSE method. With this model, we obtain predictions of settlement evolution with an error less than 10% for almost the experimental data. The model based on the KWW law describes quite well the settlement evolution in the three stages, but it shows a lack of precision in the prediction process. The KWW law is focusing on the stationary regime, when the steady state is reached. This fact induces a divergence in settlement prediction from granular compaction for materials with low values of initial packing fraction. For this reason, the proposed logarithmic model seems to be more suitable to study the settlement in coarse granular materials from a loose density state. This approach provides the estimated evolution by the proposed settlement model and an extra information about the settlement variability, taking into account the lower and upper limits of settlement values. This method can be used on track for estimating the average settlement of the sleepers taking into account train traffic parameters and

the cone penetration resistance. By considering the variability showed in settlement curves, we are able to obtain the probability to reach a threshold value of settlement on track. This probabilistic approach is of practical and fundamental interest to come to a better evaluation of the geometric degradation potential of railway track.

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